Homework #1
MATH 7360 – Fall 2023
Due: Friday, Sept. 15, 2023

Some R exercises

1. Let $a = 0.7$, $b = 0.2$, and $c = 0.1$.
   (a) Write out 0.7, 0.2, and 0.1 in binary.
   (b) In R, test whether $(a + b) + c$ equals 1.
   (c) In R, test whether $a + (b + c)$ equals 1.
   (d) In R, test whether $(a + c) + b$ equals 1.
   (e) Explain what you found. Hint: find out how addition is performed on numerics (double).

2. Create the vector $\boldsymbol{x} = (0.988, 0.989, 0.990, \ldots, 1.010, 1.011, 1.012)$.
   (a) Plot the polynomial $y = x^7 - 7x^6 + 21x^5 - 35x^4 + 35x^3 - 21x^2 + 7x - 1$ at points x_i in $\boldsymbol{x}$.
   (b) Plot the polynomial $y = (x - 1)^7$ at points x_i in $\boldsymbol{x}$.
   (c) Explain what you found.

3. Let $\boldsymbol{u} = (1, 2, 3, 3, 2, 1)^\top$.
   (a) Compute $\boldsymbol{U} = \boldsymbol{I} - (2/d)\boldsymbol{u}\boldsymbol{u}^\top$ where $d = \boldsymbol{u}^\top\boldsymbol{u}$. (This type of matrix is known as an 'elementary reflector' or a 'Householder transformation'.)
   (b) Let $\boldsymbol{C} = \boldsymbol{U}\boldsymbol{U}$, the matrix product of $\boldsymbol{U}$ and itself. Find the largest and smallest off-diagonal elements of $\boldsymbol{C}$.
   (c) Find the largest and smallest diagonal elements of $\boldsymbol{C}$.
   (d) Compute $\boldsymbol{U}\boldsymbol{u}$. (matrix times vector).
   (e) Compute the scalar $\max_i \sum_j |U(i, j)|$.
   (f) Print the third row of $\boldsymbol{U}$.
   (g) Print the elements of the second column below the diagonal.
   (h) Let $\boldsymbol{A}$ be the first three columns of $\boldsymbol{U}$. Compute $\boldsymbol{P} = \boldsymbol{A}\boldsymbol{A}^\top$.
   (i) Show that $\boldsymbol{P}$ is idempotent (in other words $\boldsymbol{P} = \boldsymbol{P}\boldsymbol{P}$) by recomputing (e) with $\boldsymbol{P}\boldsymbol{P} - \boldsymbol{P}$.
   (j) Let $\boldsymbol{B}$ be the last three columns of $\boldsymbol{U}$. Compute $\boldsymbol{Q} = \boldsymbol{B}\boldsymbol{B}^\top$.
   (k) Show that $\boldsymbol{Q}$ is idempotent by recomputing (e) with $\boldsymbol{Q}\boldsymbol{Q} - \boldsymbol{Q}$.

(l) Compute $\boldsymbol{P} + \boldsymbol{Q}$.

4. Read in the matrix in the file 'oringp.dat' on the failure of O-rings leading to the Challenger disaster. The columns are flight number, date, number of O-rings, number failed, and temperature at launch. Compute the correlation between number of failures and temperature at launch, deleting the last, missing observation (the disaster).

5. Functions

   (a) What are the three components of a function?

   (b) What does the following code return?

```
x <- 10
f1 <- function(x) {
    function() {
        x + 10
    }
}
f1(1)()
```

   (c) How could you make this call easier to read?

```
mean(, TRUE, x = c(1:10, NA))
```

   (d) Does the following function throw an error when called? Why/why not?

```
f2 <- function(a, b) {
  return(a * 10)
}
f2(10, stop("This is an error!"))
```

6. Let the $n \times n$ matrix $\boldsymbol{A}$ have elements $A(i, j) = 1/(|i - j| + 1)$.

   a Create a function that takes input argument n and output matrix $\boldsymbol{A}$.

   b Compute and print $\boldsymbol{A}$ for $n = 10$.

   c Compute and print the Cholesky factorization for $\boldsymbol{A}$ for $n = 10$. Hint: try chol() function.

   d Find the determinant of $\boldsymbol{A}$.